

Quantized Fields and Chronology Protection

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(September 21, 2000)

Abstract

Several recent possible counterexamples to the Chronology Protection Conjecture are critically examined. The “adapted” Rindler vacuum state constructed by Li and Gott for a conformal scalar field in Misner space is extended to nonconformally coupled and self-interacting scalar fields. For these fields, the vacuum stress-energy always diverges on the chronology horizons. The divergence of the vacuum stress-energy on Misner space chronology horizons cannot be generally avoided by choosing a Rindler-type vacuum state.

PACS number(s): 04.62.+v

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Over the last decade, there has been substantial interest in whether it is possible, within the known body of physical law, to create closed timelike curves (CTC) in a spacetime which is initially free of such objects [1–3]. In colloquial terms, the question is whether it is possible in principle to construct a “time machine”. The main impediment found to such a construction is the divergence of the vacuum stress-energy of quantized fields in such spacetimes [4,5]. This divergence takes place on the *chronology horizon*, the null surface beyond which CTCs first form. It is believed (but not proven) that the gravitational backreaction to such a diverging stress-energy would alter the spacetime in such a way as to prevent the formation of CTCs. The generic notion that nature will not allow the formation of CTCs is embodied in Hawking’s Chronology Protection Conjecture (CPC): *The laws of physics do not allow the appearance of closed timelike curves* [6].

Recently, a number of examples have been found of spacetimes containing CTCs in which the vacuum stress-energy tensor of a particular quantum field does not diverge on the chronology horizon [7–11]. These cases have been interpreted by some [10,11] as possible counterexamples to the Chronology Protection Conjecture, at least in the form where the vacuum stress-energy of quantized matter fields is the agent which protects chronology.

However, to be taken seriously, proposed counterexamples to the CPC should have to satisfy the same sort of criteria that are used to evaluate potential counterexamples to the Cosmic Censorship Hypothesis [12]. To begin, let us assume that the divergence of $\langle T_\mu^\nu \rangle$ on chronology horizons is in fact the mechanism of chronology protection. Then, if one finds a combination of quantized field(s), vacuum state, and spacetime such that $\langle T_\mu^\nu \rangle$ does not diverge on the chronology horizon, that combination can only be considered a valid counterexample to the CPC if:

(1) the non-divergence of $\langle T_\mu^\nu \rangle$ holds on an open set of spacetime metrics. Counterexamples must not depend on “fine-tuning” of metric parameters or topological identification scales.

(2) the vacuum stress-energy does not diverge for a collection of interacting realistic fields. Counterexamples must not depend on special field properties (e.g., being conformally

invariant, massless, or free).

Condition (1) is general; condition (2) applies only to the extent that the divergence of quantized field's vacuum stress-energy is considered to be the mechanism of chronology protection. The recent examples in which the vacuum stress-energy is regular on the chronology horizon violate one or both of these conditions.

For example, Boulware [8] and Tanaka and Hiscock [9] showed that a massive scalar field will have regular vacuum stress-energy on the chronology horizon of Grant space [13], provided the field mass is sufficiently large. In this case, one might argue that the first condition above is at least partially satisfied, since no fine-tuning of the Grant space parameters is required to render the vacuum stress-energy finite. However, the second condition is obviously violated, since not all quantized fields in the real world are massive.

As a second example, Sushkov [7] demonstrated that a complex automorphic massless scalar field would have a nondivergent vacuum stress-energy on the chronology horizon of Misner space [14] if the automorphic parameter (the angle by which the complex field is rotated upon topological identification) has a special value. This violates the first condition above, since the automorphic parameter must be fine-tuned to eliminate the divergence in the vacuum stress-energy. In addition, one could not expect all fields in nature to be free massless automorphic fields; as Sushkov points out, the addition of any interaction terms will likely restore the divergence in the vacuum stress-energy.

On the other hand, Cassidy [10] and Li and Gott [11,15] have shown that there exists a quantum state in Misner space, an “adapted” Rindler vacuum state, for which the vacuum stress-energy of conformally invariant fields is finite, in fact precisely zero, provided the Misner space identification scale is chosen to have a unique special value. They have indicated that they believe this may serve as a counterexample to the CPC, or at least to the idea that the divergence of the vacuum energy of quantized fields can protect chronology.

This “counterexample” violates the first condition above, since the vacuum stress-energy is nondivergent only for a single value of the Misner identification scale, a set of measure zero.

In this Letter, I demonstrate that the adapted Rindler vacuum also violates the second condition. I show that the (Rindler) vacuum stress-energy of a nonconformally coupled scalar field, or a conformally coupled massless field with a $\lambda\phi^4$ self-interaction will diverge on the chronology horizon for all values of the Misner identification scale. In addition, the vacuum polarization of the field, $\langle\phi^2\rangle$, diverges in all cases, even for the conformally invariant case examined by Li and Gott. Hence, the regular behaviour found by Cassidy and Li and Gott holds only for a conformally invariant, non-interacting field, and only for the stress-energy tensor. While some fields in nature (e.g., the electromagnetic field, before interactions are added) are conformally invariant, others – notably gravity itself – are not; and interactions are the rule, not the exception. All calculations are performed in the Lorentzian signature spacetime, avoiding any conceivable ambiguity associated with regularization in the Euclidean sector [15].

Misner space is constructed from Minkowski space by identifying the points:

$$(t, x, y, z) \leftrightarrow (t \cosh nb + x \sinh nb, x \cosh nb + t \sinh nb, y, z) \quad (1)$$

where (t, x, y, z) are the usual Minkowski Cartesian coordinates, b is an arbitrary positive constant, and n is an integer. The identifications take on a simpler form in Misner (equivalently, flat Kasner, or Rindler) coordinates, (η, ζ, y, z) , where

$$t = \zeta \cosh \eta, \quad x = \zeta \sinh \eta \quad . \quad (2)$$

The metric in these coordinates takes the form

$$ds^2 = -d\zeta^2 + \zeta^2 d\eta^2 + dy^2 + dz^2 \quad , \quad (3)$$

and the points which are identified are now simply

$$(\zeta, \eta, y, z) \leftrightarrow (\zeta, \eta + nb, y, z) \quad . \quad (4)$$

. The Misner space coordinates cover only the past (P , with $t > |x|$) and future (F , with $t < |x|$) quadrants of Minkowski space. There are two null surfaces $\zeta = 0$ (corresponding

to $t = x$ and $t = -x$) which are Cauchy and chronology horizons. The metric may be analytically extended across the boundaries at $\zeta = 0$; the maximal extension of Misner space is obtained by performing both extensions, although at the cost of obtaining a non-Hausdorff spacetime with a quasiregular singularity at the origin $t = x = 0$ [16]. Due to the topological identifications of Eq.(1), the extended spacetime now contains regions of closed timelike curves, namely the right ($R, x > |t|$) and left ($L, x < |t|$) quadrants of Minkowski space. In these regions, the η and ζ coordinates reverse roles, with ζ becoming a timelike coordinate and η spacelike,

$$ds^2 = -\zeta^2 d\eta^2 + d\zeta^2 + dy^2 + dz^2 \quad , \quad (5)$$

with the relation to Minkowski coordinates now being $t = \zeta \sinh \eta, x = \zeta \cosh \eta$ in R and L .

Consider now the vacuum stress-energy of a non-conformally coupled quantized massless scalar field in Misner space. The vacuum stress-energy tensor may be written in terms of the renormalized Hadamard function as:

$$\langle T_{\mu\nu} \rangle_{\text{ren}} = \frac{1}{2} \lim_{X' \rightarrow X} \left[(1 - 2\xi) \nabla_\mu \nabla_{\nu'} + (2\xi - \frac{1}{2}) g_{\mu\nu} \nabla_\alpha \nabla^{\alpha'} - 2\xi \nabla_\mu \nabla_\nu \right] G_{\text{ren}}^{(1)} \quad , \quad (6)$$

where $G_{\text{ren}}^{(1)}$ is the renormalized Hadamard function. For the adapted Rindler vacuum state considered by Li and Gott, the renormalized Hadamard function is obtained by taking the sum over Misner identifications of the image sources for the Rindler Hadamard function [17],

$$G^{(1)}(X, X') = \frac{1}{2\pi^2} \sum_{n=-\infty}^{\infty} \frac{\gamma}{\zeta \zeta' \sinh \gamma [-(\eta - \eta' + nb)^2 + \gamma^2]} \quad , \quad (7)$$

where

$$\cosh \gamma = \frac{\zeta^2 + \zeta'^2 + (y - y')^2 + (z - z')^2}{2\zeta \zeta'} \quad , \quad (8)$$

and subtracting the Minkowski vacuum Hadamard function,

$$G_{\text{ren}}^{(1)} = G^{(1)} - G_{\text{Mink}}^{(1)} \quad , \quad (9)$$

where, as usual,

$$G_{\text{Mink}}^{(1)}(X, X') = \frac{1}{2\pi^2} \frac{1}{[-(t-t')^2 + (x-x')^2 + (y-y')^2 + (z-z')^2]} . \quad (10)$$

Note that the Hadamard functions are independent of the curvature coupling of the scalar field, ξ . This is because the mode equation for the field is independent of ξ in any spacetime in which the Ricci curvature scalar vanishes, in particular, in flat space. The stress-energy tensor, defined via Eq.(6), however, is *not* independent of ξ , even in flat space.

It is a simple matter to take the required derivatives and coincidence limit of $G_{\text{ren}}^{(1)}$ to obtain the components of $\langle T_{\mu\nu} \rangle$; the result is

$$\langle T_{\zeta\zeta} \rangle = \frac{1}{3\zeta^2} \langle T_{\eta\eta} \rangle = \frac{(b^2 + 4\pi^2)[4\pi^2 - b^2 - 10(1 - 6\xi)b^2]}{1440\pi^2 b^4 \zeta^4} , \quad (11)$$

$$\langle T_{yy} \rangle = \langle T_{zz} \rangle = \frac{(b^2 + 4\pi^2)[4\pi^2 - b^2 + 20(1 - 6\xi)b^2]}{1440\pi^2 b^4 \zeta^4} . \quad (12)$$

It is now easy to see that there is no value of b for which all the components of $\langle T_{\mu\nu} \rangle$ will be regular on the chronology horizon at $\eta = 0$, unless the field is conformally coupled, $\xi = 1/6$. From Eq.(11), $T_{\eta\eta}$ and $T_{\zeta\zeta}$ will be regular at the chronology horizon only if

$$b^2 = \frac{4\pi^2}{11 - 60\xi} , \quad (13)$$

while T_{yy} and T_{zz} will be regular on the horizon only if

$$b^2 = \frac{4\pi^2}{120\xi - 19} . \quad (14)$$

Equations (13) and (14) can only be simultaneously satisfied if $b = 2\pi, \xi = 1/6$. Thus, the stress-energy of any nonconformally coupled ($\xi \neq 1/6$) scalar field will diverge on the chronology horizon in the usual ζ^{-4} manner, regardless of the particular value of the identification scale b .

Next consider a scalar field with a $\lambda\phi^4$ self-interaction, again in Misner space in the adapted Rindler vacuum state. For simplicity, only the case where the free field's vacuum stress-energy is nondivergent will be treated, i.e., a massless, conformally coupled ($\xi = 1/6$) self-interacting field. To first order in the self-coupling constant λ , the vacuum stress-energy may be written as

$$\langle T_{\mu\nu} \rangle = \langle T_{\mu\nu}^{\text{free}} \rangle + \langle T_{\mu\nu}^{\text{self-int}} \rangle , \quad (15)$$

where $\langle T_{\mu\nu}^{\text{free}} \rangle$ is the vacuum stress-energy of the free field. In Misner space, the self-interaction stress-energy is uniquely determined by the self-interaction energy density together with the requirements that the stress-energy tensor be conserved ($\langle T_{\mu}^{\nu} \rangle_{;\nu} = 0$), traceless ($\langle T_{\mu}^{\mu} \rangle = 0$, which follows as the field is conformally invariant and the spacetime is flat), and symmetric in the $y - z$ plane ($\langle T_{yy} \rangle = \langle T_{zz} \rangle$). Working in the R quadrant, these conditions yield:

$$\begin{aligned} \langle T_{\eta\eta}^{\text{self-int}} \rangle &= \zeta^2 \rho , \\ \langle T_{\zeta\zeta}^{\text{self-int}} \rangle &= -\frac{1}{\zeta} \int \rho \, d\zeta , \\ \langle T_{yy}^{\text{self-int}} \rangle &= \langle T_{zz}^{\text{self-int}} \rangle = \frac{1}{2} \left(\rho + \frac{1}{\zeta} \int \rho \, d\zeta \right) , \end{aligned} \quad (16)$$

where ρ is the self-interaction energy density. The value of ρ may be determined using the methods of Ford [19] and Kay [20],

$$\rho = \frac{\lambda}{4!} \langle \phi^4 \rangle = \frac{\lambda}{8} \langle \phi^2 \rangle^2 . \quad (17)$$

In principle, before evaluating $\langle T_{\mu\nu}^{\text{self-int}} \rangle$, the field mass M , curvature coupling ξ , self-coupling λ , and the wavefunction must be renormalized. However, the mass counterterm at first order in λ is zero for a massless field, the curvature coupling counterterm vanishes for conformal coupling at first order, and both self-coupling and wavefunction renormalization are second order in λ and hence can be ignored here. We can thus proceed to evaluate $\langle \phi^2 \rangle$ using the Hadamard function of Eq.(9),

$$\langle \phi^2 \rangle = \frac{1}{2} \lim_{X' \rightarrow X} G_{\text{ren}}^{(1)}(X, X') . \quad (18)$$

The actual evaluation of $\langle \phi^2 \rangle$, often called the “vacuum polarization” of a scalar field, is then a simple exercise, which yields

$$\langle \phi^2 \rangle = \frac{-1}{48\pi^2\zeta^2} \left[1 + \left(\frac{2\pi}{b} \right)^2 \right] , \quad (19)$$

which clearly diverges at the chronology horizons at $\eta = 0$ for all values of the Misner identification parameter b . Thus, the vacuum polarization, $\langle\phi^2\rangle$, is always divergent on the chronology horizons, even for the free field.

The self-interaction contribution to vacuum stress-energy for a self-interacting, massless, conformally coupled scalar field in the adapted Rindler vacuum to first order in λ is then, by Eqs.(16), (17) and (19),

$$\langle T_{\mu\nu}^{\text{self-int}} \rangle = \frac{\lambda}{18432\pi^4\zeta^4} \left[1 + \left(\frac{2\pi}{b} \right)^2 \right]^2 \text{diag} \left(\zeta^2, \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right) , \quad (20)$$

where the coordinates are ordered (η, ζ, y, z) .

The total vacuum stress-energy is this term plus the free-field vacuum stress-energy for the conformal massless field, which is given by Eqs.(11-12) with $\xi = 1/6$. In the special case examined by Li and Gott, with the Misner identification scale set to $b = 2\pi$, the free field vacuum stress-energy vanishes everywhere. However, the self-interaction component of the vacuum stress-energy, as calculated here to first order in λ , diverges on the chronology horizon for all values of b . The divergence has the identical ζ^{-4} dependence that is found generally. Since this calculation is done within the context of first-order perturbation theory, it is conceivable that this divergence would vanish in the full (nonperturbative) theory. It seems more likely, however, that miraculous cancellations do not occur, and thus that the divergence would persist in the full nonperturbative theory.

In conclusion, the few examples that have been found of a combination of quantized field, vacuum state, and spacetime that yield a non-divergent $\langle T_{\mu}^{\nu} \rangle$ on the chronology horizon all involve special choices of fields and spacetime properties. They are not robust when examined in a larger context. In particular, the adapted Rindler vacuum in Misner space, shown to have vanishing vacuum stress-energy for massless conformally coupled scalar field by Li and Gott, has here been shown here to have divergent stress-energy on the chronology horizons for nonconformally coupled fields and also for self-interacting fields. The adapted Rindler vacuum in Misner space does not seem to provide a convincing counterexample to the Chronology Protection Conjecture.

ACKNOWLEDGMENTS

I wish to thank Paul Anderson for helpful discussions. This research was supported by NSF grant PHY-9734834.

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